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NECESSARY CONDITIONS FOR THE EXISTENCE OF STABLE SLUGS

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Abstract—A consideration of the velocity with which the tail of a slug propagates downstream yields a minimum film height required to sustain stable slugs. Measurements of liquid heights and photographs for horizontal air/water flow in a 0.0953 m horizontal pipe provide support for this necessary condition for the existence of slugs, if the tail is considered to be an inviscid bubble. The representation of the front of a slug as a hydraulic jump yields a minimum gas velocity below which slugs cannot exist.

Key Words: slug flow, liquid heights, gas and liquid velocities

1. INTRODUCTION

Gas and liquid flowing through a horizonal pipe display a number of different interfacial configurations. At low liquid flows a stratified or a stratified/wavy regime can exist, whereby the liquid flows along the bottom of the pipe and the gas concurrently with it. An increase in the liquid velocity at a constant gas velocity causes this stratified flow to change to one of two "intermittent" patterns, plug flow or slug flow. A plug flow is found at very low gas velocities and consists of elongated gas bubbles that move along the top of the pipe. Slug flow is a stratified flow with the intermittent appearance of high-velocity liquid slugs which bridge the whole pipe and which can be highly aerated.

Considerable theoretical work has been done in recent years to understand the mechanism for the transition to intermittent flow. One approach has been to examine the stability of a stratified flow. Recent works by Lin & Hanratty (1986) and by Andritsos *et al.* (1989) have shown that such an approach appears to be valid at gas velocities which are sufficiently low that large-amplitude irregular waves generated by a Kelvin–Helmholtz mechanism (Andritsos & Hanratty 1987a, b) are not present in the stratified flow. At high gas velocities, slugs are initiated by the coalescence of Kelvin–Helmholtz waves, and not by the growth of small disturbances (Lin & Hanratty 1986).

In order to develop predictive methods for the initiation of slugs over a large range of operating and entry conditions it appears that more general necessary conditions for the existence of stable slugs have to be established. By defining the front of a slug as a hydraulic jump a minimum gas velocity is obtained. By using an inviscid model for the tail of a slug it is found that the ratio of the height of the liquid in front of the slug, h_{L1}, to the pipe diameter, D, must be above a lower limit. Photographs and measurements of h_{L1}/D for air and water flowing in a 9.53 cm horizontal pipe support the notion that the tail of the slug may be approximated as an inviscid bubble. The implication of these results with respect to the establishment of methods to predict the initiation of slugs is explored.

Simplifying assumptions, used in developing the necessary conditions, were to ignore the effects of aeration and surface tension and to assume plug flow in the body of the liquid plug and in the liquid film. This approach was pursued because it could provide a reasonable first approximation at low gas velocities where the aeration is not too vigorous, and because it was felt that not enough was known about the aeration of the slugs and velocity profiles. The good agreement of this simple model with laboratory results would seem to encourage the exploration of modifications which include the effect of aeration and which remove the assumption of plug flow by considering the viscous drag of the boundaries.

This work has largely been motivated by observations made in the Ph.D. thesis of Lin (1985). Figure 1 presents a flow regime map determined by Lin for air and water flowing in a horizontal

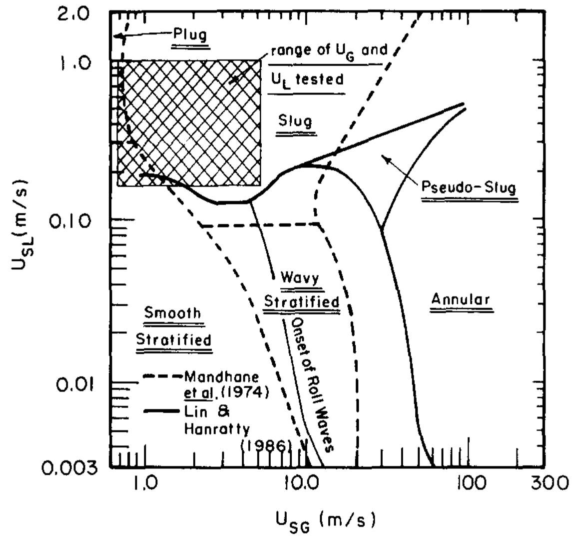

Figure 1. Flow regime map for the experimental conditions ($D = 0.0953$ m, water/air, 1 atm).

9.53 cm horizontal pipe (Lin & Hanratty 1986). Transitions from stratified to slug flow for a superficial gas velocity, U_{SG}, < 5 m/s were observed for flows that were not fully developed; i.e. the liquid was caused to flow through the pipe by hydraulic gradients, as well as the drag of the gas at the interface. For $U_{SG} > 5$ m/s the transition to slug flow occurred for a stratified layer that had irregular large-amplitude Kelvin–Helmholtz waves on it. The increase in the superficial liquid velocity, U_{SL}, required to initiate slugging when U_{SG} is increased beyond $U_{SG} = 5$ m/s can be explained by the presence of Kelvin–Helmholtz waves which cause an increase in the drag and an associated large decrease in h_L (Andritsos *et al.* 1989). In the pseudo-slug region shown in figure 1, Kelvin–Helmholtz waves coalesce to form waves that are large enough to touch the top wall momentarily; however, the conditions are such that these waves do not form slugs.

Lin (1985) observed at high U_{SG} that the height of the liquid layer over which the slugs move is not very sensitive to changes in U_{SL} or U_{SG}. This observation plus the results shown in figure 1 led him to suggest that a certain value of h_L/D must exist between slugs for the slugs to persist.

2. PHYSICAL MODEL OF A SLUG

The model of a slug, depicted in figures 2(a–d), bears similarities to the model suggested by Dukler & Hubbard (1975). As indicated in the introduction, aeration is ignored. The liquid layer in front of the slug is moving with an average velocity u_{L1} and has a height of h_{L1}. The front of the slug moves with a velocity $C_F \gg u_{L1}$, which is approximately equal to the gas velocity, if aeration effects are small (Jepson 1987a, b).

In a frame of reference moving with a velocity C_F one sees a flow pattern in the front of the slug such as sketched in figure 2(a). It resembles the sudden expansion from a small conduit to a larger sized conduit. From figure 2(a) it is seen that as the slug moves downstream it consumes liquid from the carpet in front of it at a volumetric rate

$$q_F = (C_F - u_{L1})A_{L1}, \qquad [1]$$

where A_{L1} is the area of the liquid at position 1.

Liquid consumed at the front of the slug is shed from the tail of the slug as it propagates downstream. The slug will increase or decrease in size depending on whether $q_F > q_T$ or $q_F < q_T$. The condition $q_F = q_T$ is a condition for a stationary slug to exist.

The average velocity of the liquid in the slug is designated as u_s. The length of the slug is defined as equal to that of the block of liquid that fills the whole cross section of the pipe. (The mixing vortex is included but liquid in the tail is excluded.) The flow at a position 3 may be considered uniform if the model of a sudden expansion is valid. The drag of the wall on the slug creates viscous boundary layers which grow in size downstream of position 3. If the slug is long enough these

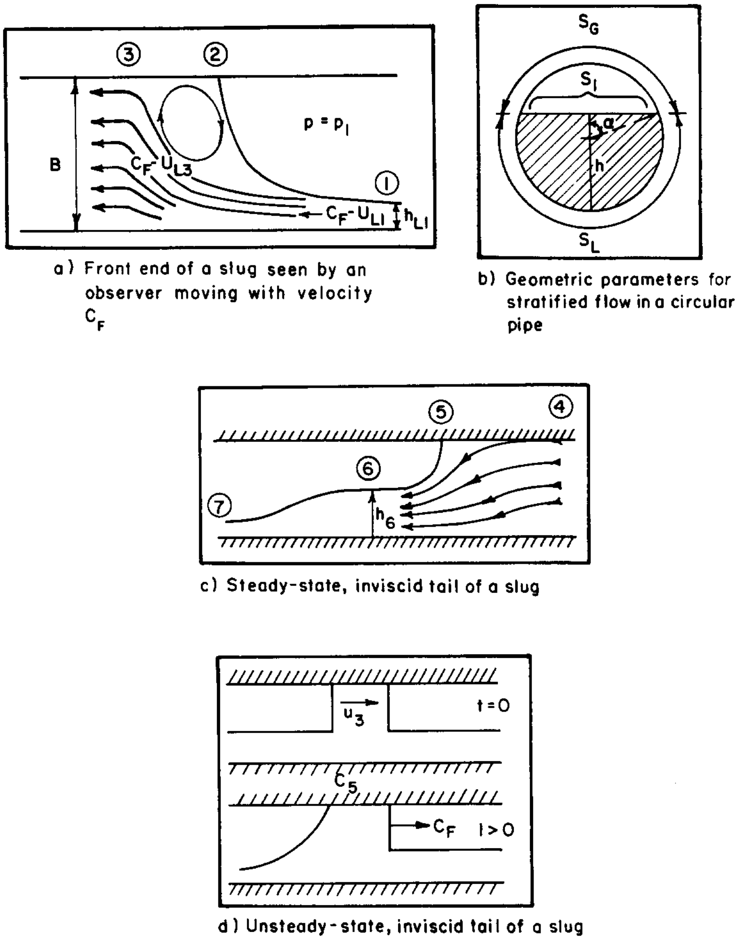

Figure 2. Physical models of the front and tail of a moving slug.

boundary layers will fill the whole conduit at the tail, but even under these conditions the flow is turbulent so that the assumption of a plug flow at position 4 is a reasonable approximation.

An appealing approach in developing a model for the tail would be to make a pseudo-steady-state assumption [such as was done by Miya *et al.* (1971) in modeling the tail of a roll wave] for which the viscous drag at the wall would be an important consideration. However, photographs of the tails of slugs (to be discussed later) do not seem to support such an assumption. In fact, the height of the liquid (at least, in the portion of the interface close to the top wall) is observed to decrease so rapidly that inertia effects should dominate effects of viscous and turbulent stresses. One might therefore expect that, to first order, the flow may be considered inviscid.

3. NECESSARY CONDITIONS DERIVED FROM THE BEHAVIOR OF THE FRONT OF A SLUG

(a) Definition of a slug

The pattern shown in figure 2(a) is not the only way the liquid interface can progress from h_{L1} to touch the top wall. If the change in height were gradual enough, a circulating zone would not be observed in the moving reference plane. Another possibility is that the liquid level increases in a staircase fashion whereby an irreversible hydraulic jump, with a height between h_{L1} and the top wall, is followed by a reversible (described by the Bernoulli equation) or an irreversible expansion (a hydraulic jump). These possibilities are discussed in detail by Ruder & Hanratty (1989). The flow pattern shown in figure 2(a) will therefore be adopted as a definition of a slug.

(b) Rectangular channel

From this definition of a slug a necessary condition can be derived from a consideration of a mass and momentum balances between positions 1 and 3, and the assumption that the flow is irreversible.

For a rectangular channel of height B the following equations are derived for an unaerated liquid which is approximated by a plug flow at stations 1 and 3:

$$(C_F - u_{L1})h_{L1} = (C_F - u_{L3})B \qquad [2]$$

and

$$\rho(C_F - u_{L3})^2 B - \rho(C_F - u_{L1})^2 h_{L1} = (p_1 - p_3)B + \frac{h_{L1}}{2}\rho g h_{L1} - \frac{\rho g B^2}{2}, \qquad [3]$$

where ρ is the liquid density, g is the acceleration of gravity, p_3 is the pressure at the top of the pipe at station 3 and $p_2 = p_1$. The elimination of $(C_F - u_{L3})$ from [3] by using [2] yields the following equation for the pressure increase:

$$\frac{(p_3 - p_2)}{\rho g B} = \frac{(C_F - u_{L1})^2}{gB}\left[\frac{h_{L1}}{B} - \left(\frac{h_{L1}}{B}\right)^2\right] - \frac{1}{2} + \frac{1}{2}\left(\frac{h_{L1}}{B}\right)^2. \qquad [4]$$

Now if the flow changes reversibly from condition 1 to 3, the Bernoulli equation is applicable and

$$\frac{(p_3 - p_2)_{rev}}{\rho g B} = \frac{1}{2}\frac{(C_F - u_{L1})^2}{gB}\left[1 - \left(\frac{h_{L1}}{B}\right)^2\right] + \left(\frac{h_{L1}}{B} - 1\right). \qquad [5]$$

By subtracting [4] from [5],

$$\frac{(p_3 - p_2)_{rev}}{\rho g B} - \frac{(p_3 - p_2)}{\rho g B} = \frac{(C_F - u_{L1})^2}{gB}\left[\frac{1}{2}\left(1 - \frac{h_{L1}}{B}\right)^2\right] - \frac{1}{2}\left(1 - \frac{h_{L1}}{B}\right)^2. \qquad [6]$$

The l.h.s. of [6] must be positive since it represents the amount of mechanical energy that is dissipated. From [6] it is therefore seen that

$$\frac{(C_F - u_{L1})^2}{gB} > 1 \qquad [7]$$

for the postulated flow to be possible. Thus the following condition is derived:

For an unaerated slug to exist in a rectangular channel it is necessary that the ratio of the difference between the slug velocity and the liquid velocity in front of the slug to the square root of the product of the gravitational constant and the height of the channel is >1.

(c) Circular pipe

A similar analysis can be applied to gas/liquid flow in a circular horizontal pipe of diameter D. The cross section of the liquid at position 1 is assumed to have the form shown in figure 2(b). The following relations are obtained from geometrical considerations:

$$\tilde{h} = \frac{h}{D}, \qquad [8]$$

$$\tilde{S}_i = \frac{S_i}{D} = 2(\tilde{h} - \tilde{h}^2)^{1/2}, \qquad [9]$$

$$\tilde{S}_L = \frac{S_L}{D} = \cos^{-1}(1 - 2\tilde{h}), \qquad [10]$$

$$\tilde{S}_G = \frac{S_G}{D} = \pi - \tilde{S}_L, \qquad [11]$$

$$\tilde{A}_L = \frac{A_L}{D^2} = \frac{1}{4}[\tilde{S}_L - \tilde{S}_i(1 - 2\tilde{h})] \qquad [12]$$

and

$$\tilde{A}_G = \frac{\pi}{4} - \tilde{A}_L. \quad [13]$$

The distance of the centroid of the hydrostatic pressure from the interface,

$$y_c = -\frac{1}{A_L}\iint y \, dA, \quad [14]$$

can be expressed as

$$\tilde{y}_c = \frac{y_c}{D} = +\frac{\sin^3 \epsilon}{12\tilde{A}_L} + \frac{\cos \epsilon}{2}, \quad [15]$$

where the void fraction

$$\epsilon = \cos^{-1}(2\tilde{h} - 1). \quad [16]$$

A mass and momentum balance on an unaerated liquid between locations 1 and 3 yields

$$(C_F - u_{L1})A_{L1} = (C_F - u_{L3})A \quad [17]$$

and

$$\rho(C_F - u_{L3})^2 A - \rho(C_F - u_{L1})^2 A_{L1} = (p_1 - p_3)A + h_c \rho g A_{L1} - \frac{\rho g A D}{2}, \quad [18]$$

where A is the pipe area and p_3 is the pressure at the top of the pipe at station 3. The elimination of $(C_F - u_{L3})$ from [18] by using [17] yields the following equation for the pressure increase:

$$\frac{(p_3 - p_2)}{\rho g D} = \frac{(C_F - u_{L1})^2}{gD}\left[\frac{A_{L1}}{A} - \left(\frac{A_{L1}}{A}\right)^2\right] - \frac{1}{2} + \frac{h_c}{D}\frac{A_{L1}}{A}. \quad [19]$$

If the flow changed reversibly from location 1 to location 3,

$$\frac{(p_3 - p_2)_{rev}}{\rho g D} = \frac{1}{2}\frac{(C_F - u_{L1})^2}{gD}\left[1 - \left(\frac{A_{L1}}{A}\right)^2\right] + \left(\frac{h_{L1}}{D} - 1\right). \quad [20]$$

Then

$$\frac{(p_3 - p_2)_{rev}}{\rho g D} - \frac{(p_3 - p_2)}{\rho g D} = \frac{(C_F - u_{L1})^2}{gD}\left[\frac{1}{2} - \frac{A_{L1}}{A} + \frac{1}{2}\left(\frac{A_{L1}}{A}\right)^2\right] + \left(-\frac{1}{2} + \frac{h_{L1}}{D} - \frac{h_c}{D}\frac{A_{L1}}{A}\right). \quad [21]$$

The solution of [21] for conditions for which the l.h.s. is zero is given in table 1, from which it is seen that for gas/liquid flow in a pipe the following necessary condition for the existence of a slug is derived:

> **For an unaerated slug to exist in a circular pipe it is necessary that the ratio of the difference between the slug velocity and liquid velocity in front of the slug to the square root of the product of the gravitational constant and the pipe diameter, $(C_F - u_{L1})/\sqrt{gD}$, must have values greater than those given in table 1 for different values of h_{L1}/D.**

(d) Necessary condition for the existence of a "Benjamin bubble"

A possible solution for an intermittent flow for the case of $(p_3 - p_2) = (p_3 - p_2)_{rev}$ can be obtained using the analysis developed by Benjamin (1968) for reversible flow past a constant pressure cavity. The case of a rectangular channel will first be considered.

The Bernoulli equation can be applied to the interface between locations 1 and 2:

$$gh_{L1} + \frac{(C_F - u_{L1})^2}{2} + \frac{p_1}{\rho} = gB + \frac{(C_F - u_{L2})^2}{2} + \frac{p_2}{\rho}. \quad [22]$$

Surface tension effects are ignored so that the pressure in the liquid at the interface equals the

Table 1. Conditions for the existence of a slug

$\frac{h_{L1}}{D}$	$\frac{(C-u_1)}{(gD)^{1/2}}$
0.1	0.9459
0.2	0.9200
0.3	0.9090
0.4	0.9108
0.5	0.9213
0.6	0.9440
0.7	0.9937
0.8	1.0778
0.85	1.1444
0.89	1.2106
0.563	0.935 (Benjamin bubble)

pressure of the gas. Therefore, $p_2 = p_1$. Furthermore, $(C_F - u_{L2}) = 0$ since 2 is a stagnation point. Equation [22] simplifies to

$$\frac{(C_F - u_{L1})^2}{gB} = 2\left(1 - \frac{h_{L1}}{B}\right). \qquad [23]$$

The use of the condition

$$\frac{(C_F - u_{L1})}{(gB)^{1/2}} = 1 \qquad [24]$$

for a reversible flow gives

$$\frac{h_{L1}}{B} = \frac{1}{2} \qquad [25]$$

and

$$\frac{(C_F - u_{L3})}{(gB)^{1/2}} = \frac{1}{2}. \qquad [26]$$

The most likely configuration for this flow is that it is the back of an elongated bubble. Similar results were also developed by Benjamin (1968) for a cylindrical pipe. From these, the conditions are obtained that

$$\frac{h_{L1}}{d} = 0.563, \qquad [27]$$

$$\frac{(C_F - u_{L3})}{(gd)^{1/2}} = 0.542 \qquad [28]$$

and

$$\frac{(C_F - u_{L1})}{(gd)^{1/2}} = 0.935. \qquad [29]$$

It is to be noted that this is a special case of the necessary conditions outlined in table 1.

4. NECESSARY CONDITIONS DERIVED FROM THE BEHAVIOR OF THE TAIL OF A SLUG

(a) General condition

Even though the velocity distribution changes from location 3 in a slug to the beginning of the tail [position 4 just upstream of the tail in figure 2(c)], conservation of mass in an unaerated liquid requires that $u_{L3} = u_{L4} = u_s$ where u_{L4} is understood to be the average liquid velocity. Therefore, a tail moving with a velocity C_T is shedding liquid at a rate equal to

$$q_T = (C_T - u_S)A_{L4}. \quad [30]$$

The condition for a stationary slug, $q_F = q_T$, is that $C_F = C_T$. The use of this condition requires the development of a relation for q_T or for $(C_T - u_s)$.

(b) Inviscid steady-state model for the tail

The steady-state inviscid solution pictures the front of the gas space behind the slug as a gas bubble that is moving with a velocity relative to the liquid slug of $(C_B - u_s)$, with $q_B = (C_B - u_s)A_{L4}$, where $q_B = q_T$ and $C_B = C_T$. The particular solution for the gas bubble that is used is that derived by Benjamin (1968).

For a rectangular channel,

$$\frac{h_{L6}}{B} = 0.5, \quad [31]$$

$$(C_B - u_s) = 0.5\,(gB)^{1/2} \quad [32]$$

and

$$q_B = 0.5\,B\,(gB)^{1/2}; \quad [33]$$

and for a circular tube,

$$\frac{h_{L6}}{D} = 0.563, \quad [34]$$

$$(C_B - u_s) = 0.542(gD)^{1/2} \quad [35]$$

and

$$q_B = 0.542\left(\frac{\pi D^2}{4}\right)(gD)^{1/2}. \quad [36]$$

From [33] and [1] the following condition is derived for an unaerated slug that is not growing:

$$\frac{(C_F - u_{L1})}{(gB)^{1/2}}\frac{h_{L1}}{B} = 0.5. \quad [37]$$

It is noted from [1] and [33] that if h_{L1}/B is less than what is given by [37] that $q_F < q_T$ and that the slug will disappear. For h_{L1}/B greater than [37] the slug will increase in length. The following condition is therefore derived:

If the tail of a slug may be approximated by a steady-state inviscid flow a necessary condition for the existence of a neutrally stable unaerated slug in a rectangular channel is that the film thickness in front of the slug be defined by [37]. The slug will grow if h_{L1} is greater than this; it will decay if h_{L1} is less than this.

The condition [31] for the liquid height right behind the tail would require an adjustment to h_{L1} by some process involving viscous effects, if the assumption of inviscid steady flow in the top part of the tail is correct. One possibility is depicted in figure 2(c), where the decrease from h_{L6} to h_{L7} is much more gradual than the decrease from h_{L5} to h_{L6}.

A relation similar to [37] can be derived for a circular tube:

$$\frac{(C_F - u_{L1})}{(gD)^{1/2}}\frac{4A_{L1}}{\pi D^2} = 0.542. \quad [38]$$

Therefore the following result is derived:

If the tail of a slug may be approximated by a steady-state inviscid flow a necessary condition for the existence of a neutrally stable unaerated slug in a circular tube is that the film thickness in front of the slug be defined by [38].

(c) Inviscid unsteady-state tail

Jepson (1987a) has proposed an inviscid unsteady-state model for the tail of a slug. He pictures the slug to originate from a rectangular block of fluid that is moving with velocity u_{L3} and is confined within retaining walls, as shown in figure 2(d). At time zero these retaining walls are removed. The front of the slug propagates downstream as a discontinuity with velocity C_F. Initially the flow at the back behaves similarly to the hydraulic analysis for the breakdown of a dam (Stoker 1957). This pictures the flow in the neighborhood of position 5 to behave as a rarifaction wave which becomes more diffuse as time proceeds. According to Stoker the velocity of this rarifaction wave at position 5 relative to fluid velocity u_s is equal to the hydraulic wave velocity for a fluid of height h_{L5}. For a rectangular channel this wave velocity equals $(gB)^{1/2}$, so that

$$(C_5 - u_s) = (gB)^{1/2} \qquad [39]$$

and

$$q_T = B(gB)^{1/2}. \qquad [40]$$

From [1] and [40] the following condition is derived:

If the tail of a slug may be approximated by an unsteady-state inviscid flow a necessary condition for the existence of a neutrally stable unaerated slug in a rectangular channel is that the film thickness in front of the slug be defined by the equation

$$\frac{(C_F - u_1)}{(gB)^{1/2}} \frac{h_{L1}}{B} = 1. \qquad [41]$$

For a circular pipe, the wave velocity would equal $(gh_{ef})^{1/2}$. For the case of a full pipe, $h_{ef} = \pi D/4$ (Jepson 1987a). Then

$$(C_5 - u_s) = (gh_{ef})^{1/2} = \left(\frac{g\pi D}{4}\right)^{1/2} \qquad [42]$$

and

$$q_T = \frac{\pi D^2}{4}\left(\frac{\pi}{4}\right)^{1/2}(gD)^{1/2}. \qquad [43]$$

From [43] and [1] the following condition is derived:

If the tail of a slug may be approximated by an unsteady-state inviscid flow a necessary condition for the existence of a neutrally stable unaerated slug in a circular pipe is that the film thickness in front of the slug be defined by the equation

$$\frac{(C_F - u_{L1})}{(gD)^{1/2}} = \frac{\pi D^2}{4A_{L1}}\left(\frac{\pi}{4}\right)^{1/2}. \qquad [44]$$

5. IMPLICATIONS OF DEFINED NECESSARY CONDITIONS

The necessary conditions for the existence of slugs in a circular pipe derived from [21] and the steady-state inviscid model for [38] are plotted in figure 3. The region where these relations predict it is possible for stable slugs to exist is to the right of curve 1 and above curve 2. The lower limit of this region is the height of the liquid layer between stable slugs. The slugs will increase in length or decrease in length depending on whether h_{L1} is greater or less than this lower limit.

The stability curve calculated by Lin & Hanratty (1986) for a 9.53 cm pipe for air/water at atmospheric pressure ($\rho_G/\rho_L = 1.12\text{–}10^{-3}$) is also shown in figure 3 as curve 3. Experimental observations indicate that at $h_{L1} < ca\ 0.3D$ the instability produces Kelvin–Helmholtz waves and does not lead to an intermittent flow. The stability curve is, therefore, terminated at this point. It is noted that the necessary condition derived from the inviscid steady-state model for the tail does not rule out the possibility of the growth to slugs of small-amplitude disturbances generated

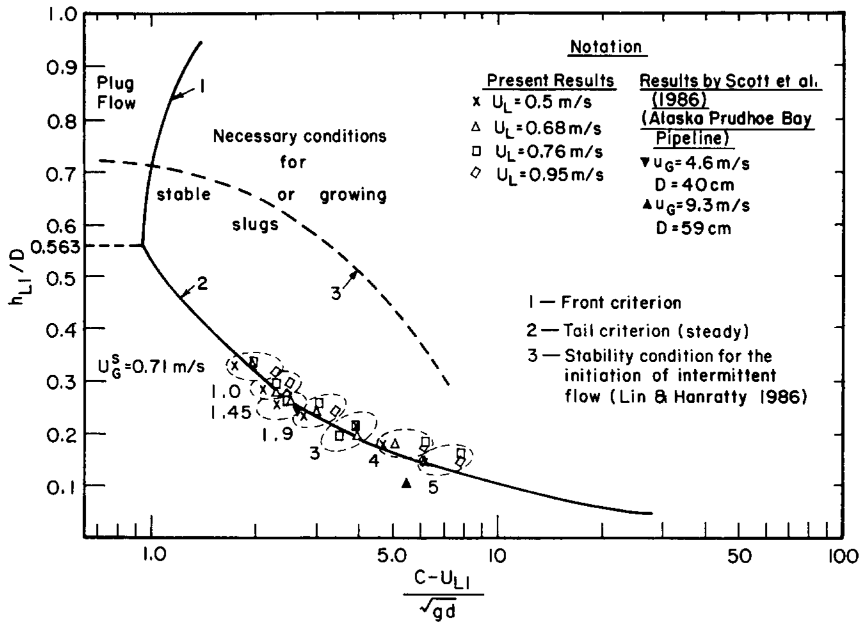

Figure 3. Theoretical necessary conditions for the existence of slugs and experimental results.

at the interface by the air flow. In fact, figure 3 seems to suggest the possibility of initiating slugs at lower liquid heights than predicted by linear stability theory by introducing large disturbances at the pipe inlet.

6. VISUAL OBSERVATIONS

Figures 4(a, b) show photographs of the fronts of liquid blockages in a 0.0953 m pipe under conditions of mild and strong aeration. It is noted that the front of the blockage for $U_{SG} = 1.5$ m/s and $U_{SL} = 0.5$ m/s [in figure 4(a)] resembles the hydraulic jump modeled in section 3, except for the appearance of air bubbles formed as the high-speed liquid front moves across the slower-moving stratified flow. These bubbles concentrate in the circulating zone. It is noted that this photograph supports the observations made by Jepson (1987b) in his experiments with stationary hydraulic jumps that the bubbles break away in waves. These bubbles eventually rise to the top of the pipe and can even form a long bubble or a layer of gas before moving out the tail of the liquid slug.

The conditions in figure 4(a) are rather mild and represent one of the first slugs seen in the pipe before the flow reached a stationary condition. The degree of aeration increases with an increasing gas velocity and an increasing frequency of slugging. For example, figure 4(b) is a photograph for $U_{SG} = 1.45$ m/s and $U_{SL} = 0.95$ m/s after a stationary condition was reached with a large number of slugs in the pipe. A layer of foam appears to precede the jump. Figure 4(c) shows the main body of the slug for a stationary flow at $U_{SG} = 1.65$ m/s and $U_{SL} = 0.95$ m/s. A thin layer of air can be seen on the top of the pipe.

Slug tails observed in the 0.0953 m pipe are shown in figures 5(a, b). The basic assumption of the steady-state model in section 4(b) is that the air space at the top end of the tail resembles a gas bubble. The particular model chosen for this bubble is the inviscid solution of Benjamin (1968). Figure 5(a) would seem to support this notion only partially. According to the Benjamin model the stagnation point of the bubble is at the wall, whereas the bubble shown in figure 5(a) has its nose displaced slightly downward from the wall. Figure 5(b) shows the tail of a slug which was more highly aerated than the one shown in figure 5(a). Again the upper part of the tail is roughly approximated by a bubble. However, it is to be noted that the bubble is distorted by the release of air from the tail at the top pipe wall. The values of h_{L6} indicated in the photos seem to vary depending on how air is being released. On average, there is rough agreement with the prediction for a Benjamin bubble, $h_{L6}/D = 0.563$.

Figure 5(c) shows a photograph (taken by Nikolaos Andritsos) of an entire slug for air and water flowing in a horizontal 2.54 cm pipe. The aeration is so vigorous that a layer of foam precedes the slug to give a staircase appearance. (This type of behavior has been observed in studies of stationary slugs carried out in our laboratory. The introduction of a defoaming agent in these experiments was found to change the appearance of the front of these stationary slugs.) Again the tail of the slug is seen to have the shape of a bubble. The bubbles that rise to the top of the tube in this particular slug keep an approximately spherical shape and do not form an elongated bubble or a gas layer.

These photographs suggest that for conditions in the cross-hatched area in figure 1, the back of a slug can be approximated by a bubble. The necessary condition derived for an inviscid steady-state tail would, therefore, seem to be more appropriate than that derived for an unsteady tail.

It is clear from these photographs that the neglect of aeration in developing necessary conditions for the existence of slugging could introduce errors in the analyses presented in sections 3 and 4. For the case of moderate aeration this simplification might not be too serious in developing hydraulic jump equations for the front since overall balances are written between the liquid carpet, which is not aerated, and a location behind the circulating zone, which is sparsely aerated. However, this might not be the case at high gas velocities (say $U_{SG} = 5$ m/s) for which the slug body is highly aerated. The principal influence of aeration in modeling the tail appears to be associated with the passage of air out of the top of the tail. Clearly, the use of the Benjamin model is only an approximation.

7. MEASUREMENTS OF THE HEIGHT OF THE LIQUID CARPET

(a) Description of the experiments

Measurements of the height of the liquid carpet over which the slugs move were done in a horizontal transparent plastic pipe 24.6 m long and 0.0953 m in diameter. Air and water were mixed at an inlet tee through which the air entered from the top branch. The mixture discharged in a 0.46 m dia horizontal separator, which was at atmospheric pressure. Because of the sensitivity of the flow pattern to pipe inclination (Barnea *et al.* 1980) great care was taken to level the pipe horizontally.

The height of the liquid was measured by a conductance method which relates the electrical conductance between two parallel-wire electrodes to the height of the liquid in which they are immersed. The method, which is based on techniques developed by Swanson (1966), was used in a number of previous studies from this laboratory (Kim 1971; Miya 1970; Lin 1985; Andritsos *et al.* 1989). A good discussion of the theory associated with this method is contained in a publication by Brown *et al.* (1978). In this particular application two stainless steel wires, 0.75 mm dia and with a separation distance of 1.5 mm, were strung vertically across the diameter of a test section located 190 pipe diameters from the inlet. A second pair of wires of identical dimensions was located 0.267 m downstream. The two probes were interfaced with an electronic multichannel analyzer that operated by sending an oscillating signal to the probes and converting the response to an analog voltage signal. This output was passed through an amplifier with an adjustable gain of up to 10^5. The output data were digitized at 100 points/s and stored on the magnetic disc of a computer.

The two probes were used principally to measure slug velocities. The time required for a slug to travel along the distance between the two probes was calculated using the method of cross-correlation described by Lin (1985). This method is based on obtaining a maximum in the cross-correlation function of the two voltage signals at a characteristic time delay. This represents an average of the time delays of all the slugs in the record for fixed flow conditions.

The probes were calibrated under static conditions before and after each set of experiments by filling the test section to a known level with tap water. The range of flow conditions which was tested in the course of the present experiments is represented by the cross-hatched area in figure 1. The superficial gas velocities varied from 0.71 (which is close to the Mandhane slug/plug transition) up to 5 m/s. The maximum superficial liquid velocity tested was 0.95 m/s.

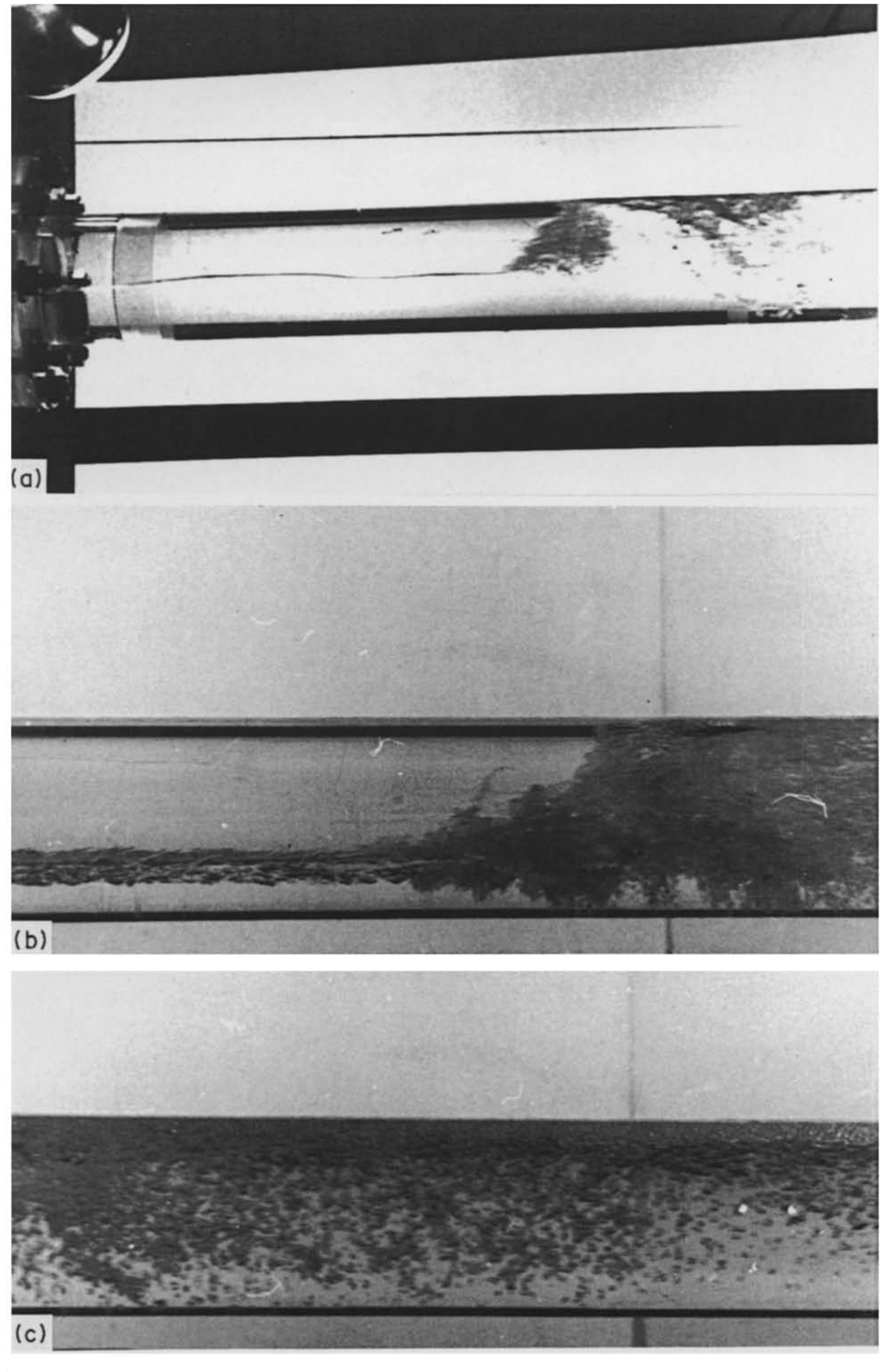

Figure 4. Photographs of typical slug fronts and body. (a) The front of a moving slug; $U_{SG} = 1.5$ m/s, $U_{SL} = 0.5$ m/s. (b) The front of a moving slug; $U_{SG} = 1.45$ m/s, $U_{SL} = 0.95$ m/s. (c) The aerated body of a moving slug; $U_{SG} = 1.45$ m/s, $U_{SL} = 0.95$ m/s.

(b) Liquid height variations

Height tracings at 18.1 m from the inlet are shown in figure 6 for a condition that corresponds to a liquid flow slightly larger than that required to initiate slugs, $U_{SL} = 0.22$ m/s and $U_{SG} = 1.30$ m/s. The dashed line represents the height of the liquid layer that is predicted by linear theory for the initiation of slugs. It is noted that after the passage of the slug the liquid layer behind

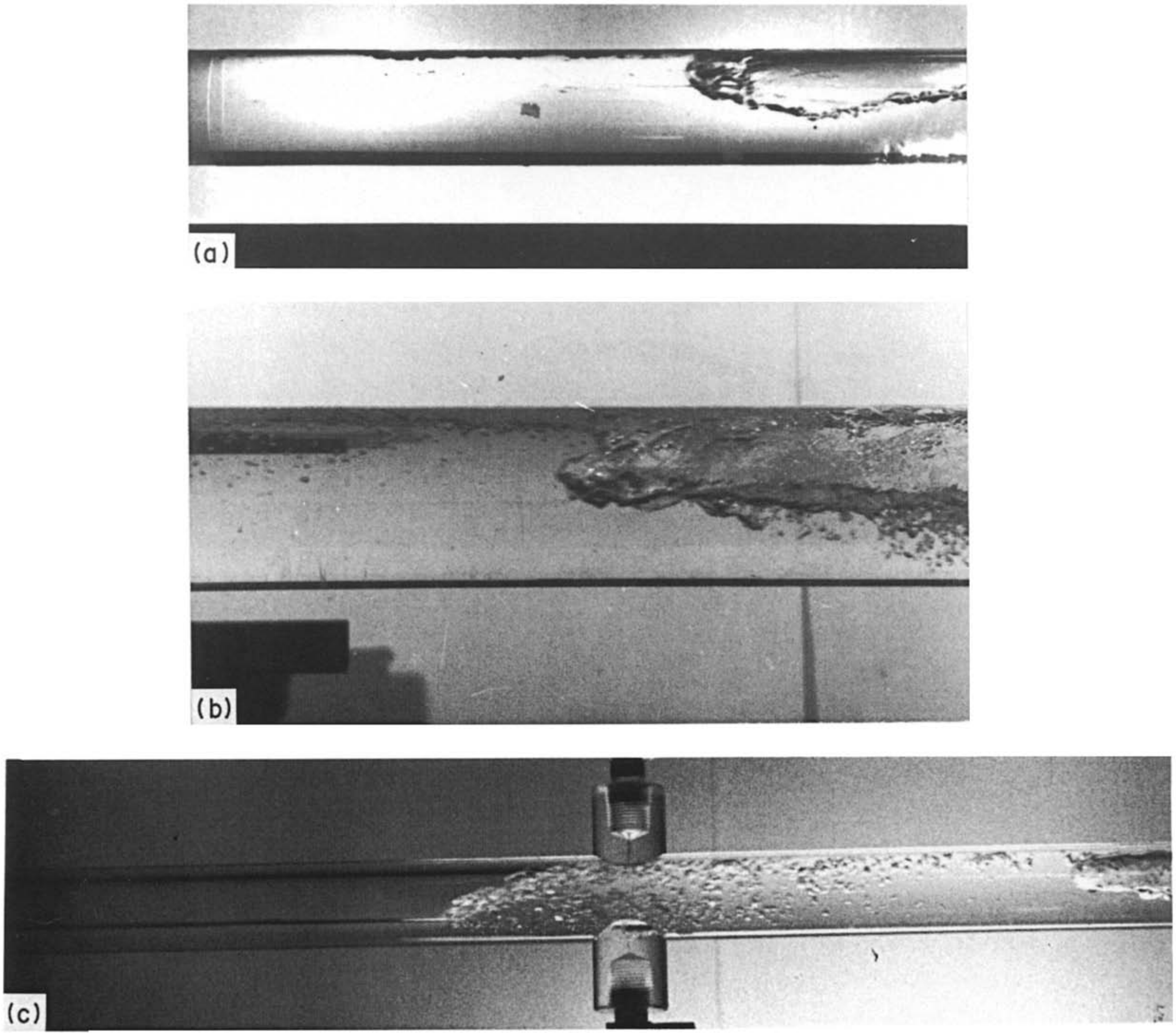

Figure 5. Photographs of typical slug tails. (a) Typical slug tail corresponding to the "Benjamin bubble" model ($U_{SG} = 1.45$ m/s, $U_{SL} = 0.3$ m/s). (b) Instantaneously distorted tail ($U_{SG} = 1.45$ m/s, $U_{SL} = 0.95$ m/s). (c) A typical slug in a 0.0254 m i.d. horizontal pipe.

the slug continues to decrease because the air flow thins out the film. Eventually new liquid from the inlet in the form of a flooding wave reaches the location of the probes and the liquid again starts to increase. The liquid level increases to a point where an instability triggers another slug.

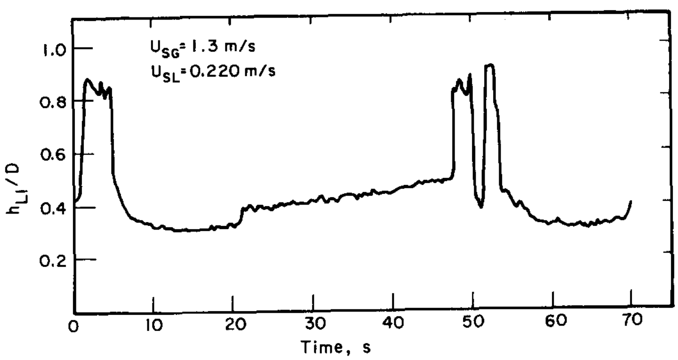

Figure 6. Liquid height tracings for $U_{SG} = 1.3$ m/s; $U_{SL} = 0.22$ m/s.

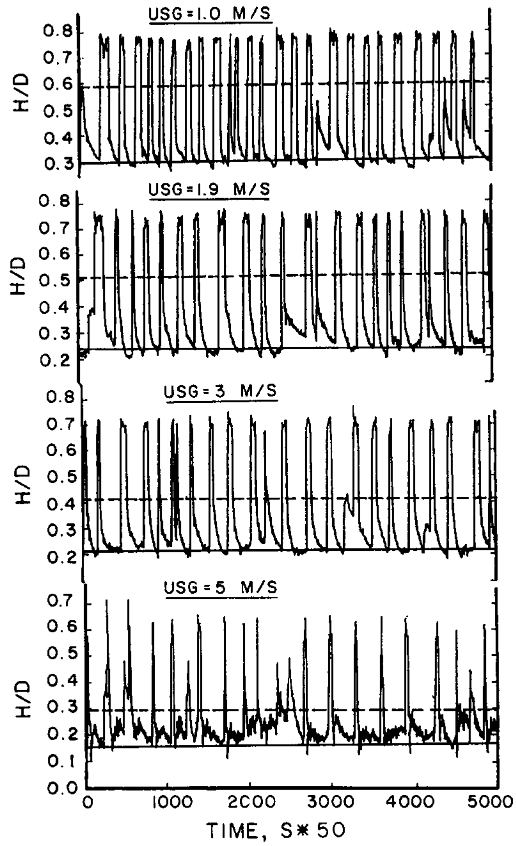

Figure 7. Liquid height tracings for high U_{SL} (0.95 m/s), different U_{SG}.

Since large hydraulic gradients exist it is quite likely that the slug is initiated a considerable distance upstream where the height is close to that represented by the dashed line. It is noted that a pair of slugs can also be formed at different locations in the pipe with the thickening of the liquid.

The slugs shown in figure 6 move faster than the flooding wave. According to the theory developed in section 4 they will increase in length. Eventually they catch up with the tail behind the first slug and will stop growing or, even, decrease in size. As the liquid flow rate increases, the number of slugs in the pipe at any one time increases and the slug pattern adjusts at this location in the pipe so that there is a well-defined carpet over which the slugs move. This is illustrated by liquid height measurements at 18.1 m from the inlet, shown in figure 7, for a high liquid superficial velocity and for several superficial gas velocities. For these flow conditions, the initiation of slugs was observed when the stratified flow reached the value of h_L/D depicted by the dashed lines. It is noted here, as well as in figure 6, that h_L/D does not reach 1.0 with the passage of a slug. This is probably associated with the aeration of the slug. For example, a voltage from the conductivity probes corresponding to $h_L/D = 0.75$ could indicate a layer of gas on the top of the slug and an unaerated liquid below it with $h_L/D = 0.75$. However, it could correspond to a uniform distribution of air in the liquid with a void fraction of about 19%. The most likely interpretation is something between these two extremes.

The liquid carpet over which the slugs move is not aerated so its height is believed to be correctly represented by the tracings. The solid lines in figure 7 represent the values of h_L/D chosen to represent the carpet for these runs. The dashed lines represent the values of h_L/D required for the initiation of intermittent flow by a hydrodynamic instability for the same flow conditions (Lin & Hanratty 1986).

The slugs were moving too fast for liquid height measurements, such as those shown in figures 6 and 7, to capture the detail shown in the photographs. Thus, they could not be used to see whether [34] approximates the liquid height just behind the bubble.

8. HEIGHT OF LIQUID IN FRONT OF A SLUG

(a) Measurements

The average height of the liquid in front of fully developed slugs was determined from results of the type shown in figures 6 and 7. Only cases in which more than one slug existed in the pipe were considered, so as to eliminate situations where the slug is preceded by a flooding wave. The experiments indicated that the value of h_{L1} was not very sensitive to changes in the liquid throughput. This is clearly seen in the examples in figure 8. It was also noted that h_{L1} decreased with increasing U_{SG}. Both these trends are seen in figure 9 where all measurements of h_{L1} are summarized.

(b) Comparison with theory

The existence of a minimum liquid film height below which slugs will decay suggests that a slug pattern will tend to approach a fully developed condition with a well-defined minimum liquid height. It is therefore of interest to compare the measurements in figure 9 with [38].

This is done in figure 3. The evaluation of the abscissa, corresponding to the height measurements, requires the slug velocity and the liquid film velocity, u_{L1}. The slug velocity was obtained from the measurements shown in figure 10. The liquid velocity in the carpet was calculated from u_G (assumed equal to C) and the measured h_{L1} by using the relations developed by Andritsos & Hanratty (1987a, b) for a wavy stratified flow. It turns out that for the conditions considered $C \gg u_{L1}$, so that errors in estimating u_{L1} are not serious.

For unaerated slugs, C equals the gas velocity. It is of interest to compare the actual C (approximated by u_G in this study) used in the comparison in figure 3 with estimates obtained from the literature. Figure 11 gives two such comparisons. These show good agreement between measured slug velocities and the correlation of Zuber & Findley (1965) and between measured slug velocities and gas velocities calculated from the void correlation of Hughmark (1962), using

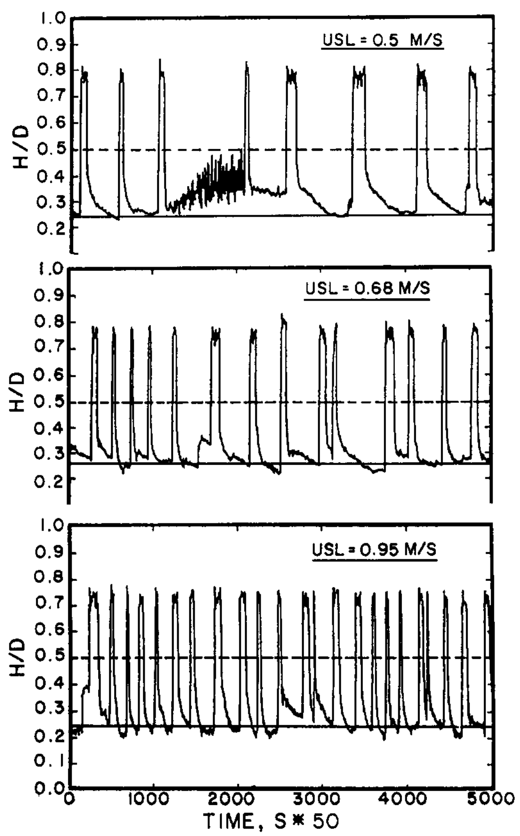

Figure 8. Liquid height tracings for different U_{SL}, constant $U_{SG} = 1.9$ m/s.

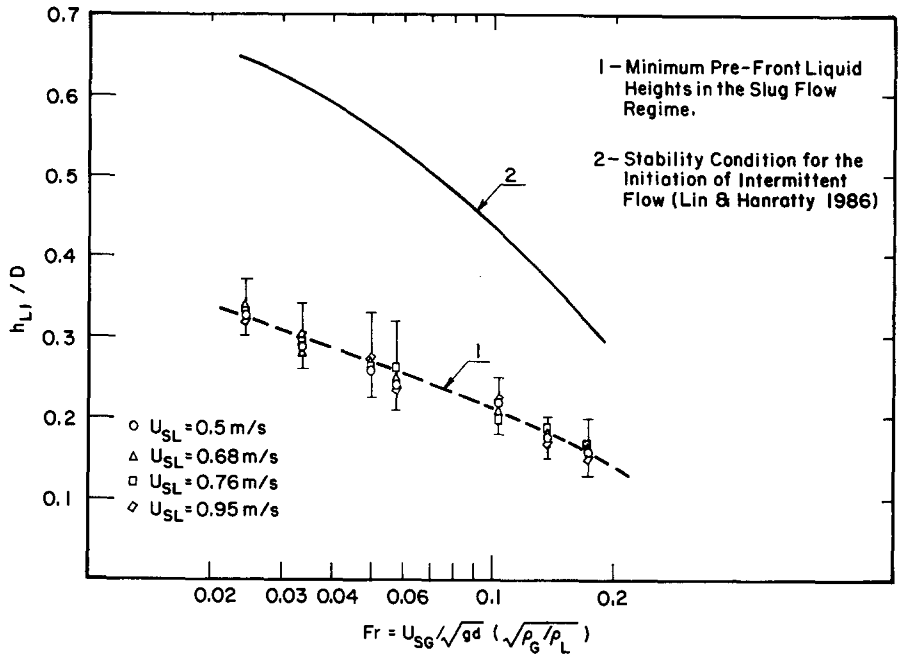

Figure 9. Minimum liquid heights measured in the slug flow regime.

$u_G = U_{SG}/\epsilon$. The latter supports the assumption that u_G is close to C for the present experimental conditions and agrees with earlier measurements by Lin & Hanratty (1987). Figure 11 also shows the variation of voidage for the superficial liquid and gas velocities tested. Here, the void fraction, ϵ, was calculated as C/U_{SG}, where C is the result of the present measurements of slug velocity. Again the agreement with the prediction by Hughmark appears to be satisfactory.

As mentioned earlier the dashed curve in figure 3 represents conditions required for the initiation of slugs from small-amplitude disturbances for air/water flow in a 0.0953 m pipe. The figure therefore documents what is already shown in figures 7–9 that for this flow the carpet height is less than the height predicted from stability theory.

It is noted from figure 3 that the measured carpet heights are reasonably close to the necessary condition for the existence of slugs [38] developed in section 4. A closer examination reveals that for a fixed superficial gas velocity the measurements deviate more and more from the curve with increasing liquid flow. Since the degree of aeration was observed to increase with increasing liquid

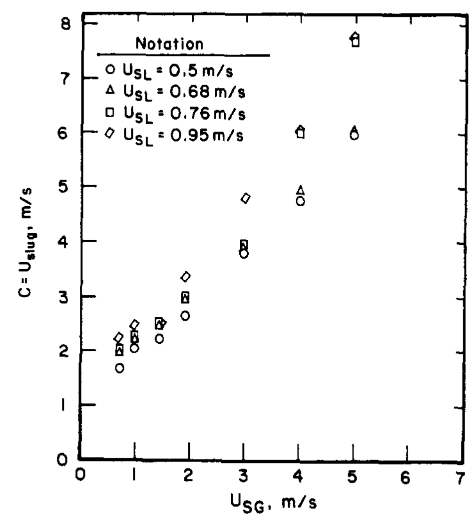

Figure 10. Slug velocities measured by means of two parallel-wire probes.

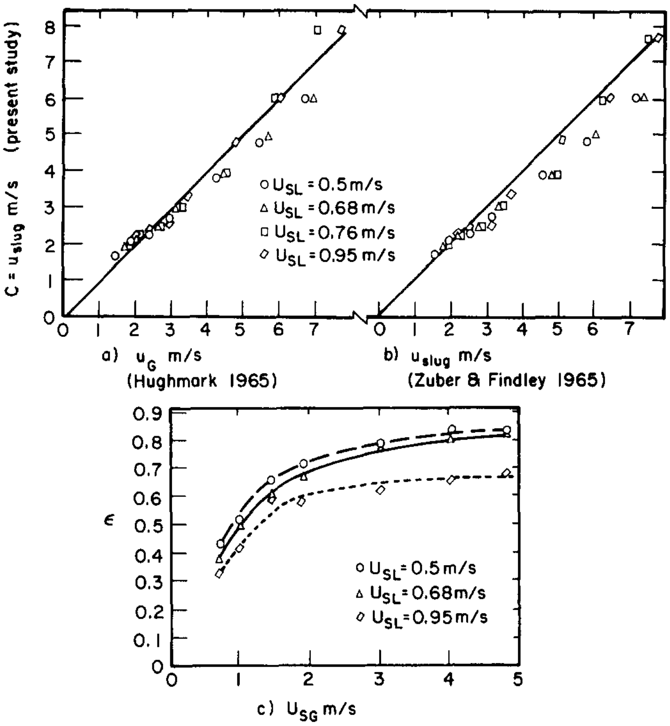

Figure 11. Comparison of measured slug velocities with theoretical predictions (a and b) and calculated void fraction values in the slug flow regime (c).

flow, this suggests that the effect of aeration is to displace the data to the right. Necessary condition [38], derived for an unaerated slug, thus appears to provide a low bound for h_{L1}/D.

Results of Scott *et al.* (1987), obtained from the Prudhoe Bay Field of the Alaska pipeline, are also shown as the two darkened points in figure 3. These were obtained in 0.4 and 0.59 m i.d., 4.5×10^3 m long sections of horizontal pipes at actual gas velocities of 9.3 and 4.6 m/s. Minimum h_L/D values of 0.115 and 0.25 were reported. In these cases, the values of the actual gas velocities were estimated from the reported locations of the same slugs at different time periods. Pre-front liquid velocities were calculated by means of the procedure recommended by Andritsos & Hanratty (1987a, b). The conditions for these tests are quite different from the conditions for air/water flow in a 0.0953 m pipe. Consequently, the stability curve in figure 3 is not applicable. However, the solid curve should be valid for all conditions. Reasonable agreement between the Alaska pipeline data and [38] is again noted.

It should also be noted from figure 3 that both the present results and the results of Scott *et al.* appear to correspond to Froude numbers higher than those given in table 1, which seems to support the necessary condition for the existence of slugs developed in section 3 from the hydraulic jump concept for slug fronts.

9. COMPARISON OF NECESSARY CONDITION [38] WITH SLUG INITIATION RESULTS

The results in figure 3 correspond to situations in which linear stability predicts the initiation of slugs from a well-formed stratified flow. It is noted in these figures that in all of these cases the critical h_{L1}/D is much larger than the h_{L1}/D predicted as a necessary condition for the existence of slugs.

An interpretation of a minimum h_{L1}/D for the liquid in front of stable slugs is that slugs initiated at the pipe inlet will not decay if the liquid height exceeds this value. This raises the question whether slug flow can be sustained at lower liquid throughputs if a highly disturbed entry is used. Work by Salkudean *et al.* (1983) supports this notion. They used local decreases in the pipe cross

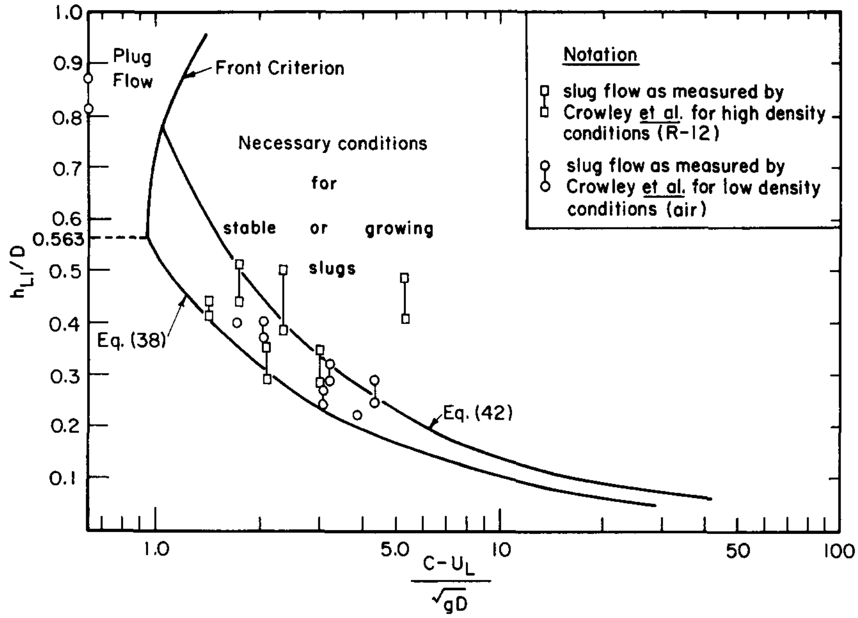

Figure 12. Experimental results by Crowley *et al.* (1986) vs theoretical necessary conditions for the existence of stable slugs.

section and peripheral obstructions and found slug flows at superficial liquid velocities at about 60% of that observed for an undisturbed entry. This same type of result has also been obtained by Bendiksen & Malnes (1987).

In figure 12, the results reported by Crowley *et al.* (1986) are compared with the stable slug condition, i.e. [38]. Crowley *et al.* carried out experiments with water and two different types of gas: air (low density case) and R-12 (high density case) in a 0.178 m i.d. horizontal pipe. They reported the flow parameters, including gas and liquid carpet actual velocities and liquid heights measured during the experiments. In figure 12, the minimum liquid heights and corresponding Froude numbers are plotted for the cases where the stable slug flow was observed to occur in the pipe. As is seen, the results typically fall in the proximity of curve 2.

10. CONCLUSIONS

Two necessary conditions are developed for the existence of stable slugs. One of these is derived by defining the front of a slug by a hydraulic jump. It gives a gas velocity below which slugs cannot exist.

The second condition, which is derived from the propagation velocity of the tail of the slug, defines a minimum height of the liquid layer over which stable slugs can propagate. Liquid height measurements and photographs of slugs support the suggestion that the tail may be approximated by a Benjamin bubble.

For air/liquid flows at atmospheric conditions the minimum h_L/D predicted by this criterion is much lower than the h_L/D predicted by a stability analysis of a stratified flow. This supports the notion that stable slug flow could be generated at lower liquid flow rates with a disturbed entry.

It is interesting to note, however, that this criterion for a minimum h_L/D is independent of gas density and that instability of a stratified flow occurs at lower gas velocities for higher gas densities. This suggests that under high-pressure conditions the h_L/D required for growth of small disturbances will be less than the minimum h_L/D derived from a tail velocity.

One of the chief limitations of the necessary conditions presented is the assumption of an unaerated slug. The approximate agreement of predictions with experimental results would seem to encourage the exploration of the influence of aeration. This could be particularly important in the consideration of slugs at high gas velocities.

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